

QUESTION: To open a lock, a key is taken out of a collection of n keys at random. If the lock is not opened with this key, it is put back into the collection and again a key is tried. The process is repeated. It is given that with only one key in the collection, the lock can be opened. The probability that the lock will open in n trials is

OPTIONS:

- a) $\left(\frac{1}{n}\right)^n$
- b) $\left(\frac{n-1}{n}\right)^n$
- c) $1 - \left(\frac{n-1}{n}\right)^n$
- d) $\frac{(n-1)^{n-1}}{n^n}$

SOLUTION: Suppose that the lock opens in n th trial. Therefore the probability of success in n th trial is $\frac{1}{n}$ also the probability of failures 1 to $(n-1)$ th trial

is $\left(1 - \frac{1}{n}\right)^{n-1}$. Hence the probability that the lock will open in n trial is

$$\left(1 - \frac{1}{n}\right)^{n-1} \times \frac{1}{n} = \frac{(n-1)^{n-1}}{n^n}$$